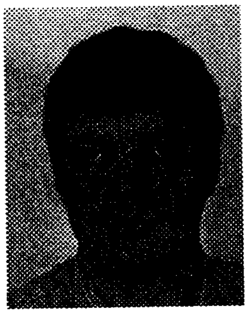
PACKING CALCULATIONS AND CONCRETE MIX DESIGN



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ABSTRACT

This article deals with computer-based calculations of packing of the aggregates as an aid when chosing a concrete mix design. The packing theory and the developed computer program is presented. Furthermore, an investigation relating packing of the aggregates and the properties of the fresh concrete is described. Finally, application of the packing calculations and practical experience is discussed.

Key words: Concrete mix design, packing, computer program, practical experience.

1. INTRODUCTION

Concrete mix design involves choice of accessible types and amounts of materials with the purpose of fulfilling the demands of the contractor to the properties of the fresh concrete as well as the demands of the building owner to the properties of the hardened concrete. Everything under considerations of economy.

When chosing a concrete mix design, it is always desirable to compose the aggregates as densely as possible. That minimizes the necessary amount of binder which has to fill the cavities between the aggregates. Apart from an obvious economic benefit, a minimum amount of binder in a concrete results in less shrinkage and creep and a more dense and therefore probably a more durable and strong concrete.

Also the fine particles in a concrete, i.e. the cement and microfiller materials, can be composed according to the principle of packing. This leads to a reduction in the water demand. Composition of the fine particles according to the principle of packing is especially important for "modern" concretes with a low water-cement ratio, with additions (fly ash, microsilica etc.) and with plasticizing additives. However, this is outside the scope of this article.

1.1 Historical background

The concept of particle packing is not new. Already in 1907 Fuller and Thompson, /1/, investigated the importance of the size distribution of the aggregates and the properties of the concrete on the basis of packing of the constituent materials. The so-called Fuller size distribution was the result of these considerations. The Fuller size distribution indicates the composition of the aggregates which theoretically, under more or less realistic assumptions, leads to an optimal packing.

Suenson, /2/, presented in 1911 experimentally based diagrams of packing of the aggregates. These diagrams look like the triangular packing diagrams, which are the result of the computer-based packing program in this article, see FIG. 2 in section 3. Powers, /3/, also has to be mentioned for his extensive work with regard to concrete mix design on the basis of packing.

More recent, Bache, /4/, has been arguing for the use of the concept of packing for concrete mix design.

1.2 Current approach

This article treats computer-based packing calculations as an aid when chosing a concrete mix design. Only aspects relating to packing of the aggregates are treated. Theoretically, there is an infinite number of possibilities for composing concrete. Practically, it is impossible to estimate the effect of all the possibilities. This is the background for the look for help in packing models, which can calculate the packing of any combination of materials. The advantage by using the packing concept is that it is based on a correct physical mechanism in contrast to conventional methods of concrete mix design.

The packing theory behind the model, as well as use of the developed computer model, is described. Furthermore, an investigation relating packing of the aggregates and the properties of the fresh concrete is described. Finally, application of the packing calculations is discussed together with obtained practical experience.

2. THE PACKING THEORY

The computer program is based on a model developed in the light of the principle of packing of binary mixtures and exended to multi-component mixtures, Stovall et al. /5/. The basic model is developed further and modified by incorporating an experimentally determined packing, Glavind et al. /6/.

Packing can be defined as the volume of particles in relation to the total volume or as one minus the porosity. The mono-disperse packing, α, can be defined as the packing of equally sized particles.

The mono-disperse packing is an important parameter in the packing calculation. For spherical particles, the mono-disperse packing equals 0.60-0.64. However, the shape of the aggregates - as well as the shape of other natural occurence materials - is not spherical. Therefore, the mono-disperse packing is normally less than 0.60-0.64. It is practically impossible to determine the mono-disperse packing experimentally, and the following procedure, which is introduced in Glavind et al. /6/, can therefore be followed.

For each material which is investigated in the packing analysis, the packing is determined experimentally, see section 4. The size distribution for each material is divided into a sufficient amount of fractions and the mono-disperse packing is determined by iteration so that the experimentally determined - and the theoretically calculated - packing agrees.

Note that the mono-disperse packing for a material always will be less than, or equal to, the packing of the material. If a material consists of particles with one size, the mono-disperse packing is equal to the packing. On the contrary, if the size distribution is wide, the difference between the mono-disperse packing and the packing is large.

When a binary mixture (a mixture with two particle sizes) is considered, either the small or the large particles are packed as densely as possible with the mono-disperse packing. The two situations result in two different values for packing. It can be shown that the smallest value is the correct value. The described principle is valid for a multi component mixture, too. Equation (1) shows calculation of the packing under the assumption that the i'th size fraction is packed as densely as possible. ϕ is the volume fraction of each fraction.

$$Packing=\alpha_i+(1-\alpha_i)\sum_{j=1}^{i-1}\phi_j+\sum_{j=i+1}^{n}\phi_j \tag{1}$$

The model takes into account two different interaction effects.

- 1) The "wall" effect, which is explained in the following way. Small particles close to a larger particle (or the wall of a container) can not be packed as dense (with the mono-disperse packing) as in the bulk.
- 2) The other interaction effect appears when the small particles are so large that they can not fit in between the cavities between the large particles without disturbing the packing of the large particles. This effect is characterized by a so-called μ-value. The μ-value states the maximum size ratio between two particle sizes which allows the small particles to pack in between the large

particles without disturbing the large particles.

If the last mentioned effect is described with the function f(i,j) and the "wall" effect by the function g(i,j), equation (1) can be modified to:

$$Packing=\alpha_i+(1-\alpha_i)\sum_{j=1}^{i-1}g(i,j)\phi_j+\sum_{j=i+1}^{n}f(i,j)\phi_j \qquad (2)$$

For a more detailed description of the formulas, the reader is referred to Stovall et al., /5/.

3. THE COMPUTER PROGRAM

The procedure described in section 2 has been translated into a computer program which is now commercially available at the Concrete Center, Danish Technological Institute (for DKK 12,000.- excl. VAT). The input and the output to the program is shown in TABLE 1. There is input for each material in the packing analysis and for each packing calculation.

Table 1. *Input and output to the packing program*

	Input	Output
Material	• Density • Grading curve • Experimental packing	• Mono-disperse packing
Calculation	• μ-value • Amount of divisions of the grading curve • Amount of calculation combinations	• Packing diagram for a two-component system • Packing diagram for a three-component system • Compound grading curve

The density and the grading curve do not need explanation. Measurement of packing is treated in section 4. The μ-value and the mono-disperse packing is described in section 2. The amount of divisions of the grading curve and the amount of calculation combinations determines the accuracy of the calculation. The user manual to the program, Olsen et al. /7/, describes in detail how to use the program.

FIG. 1 shows a packing diagram with two materials and FIG. 2 a packing diagram with three materials. The result in FIG. 2 is shown as contour lines in %. The marked point at the figure has a packing density of 84 % and the corresponding material composition is 35 % Sand 0-4 P, 25 % Gravel 6-16 P and 40 % Gravel 16-32 P. FIG. 3 shows the compound grading curve for the marked point at FIG. 2.

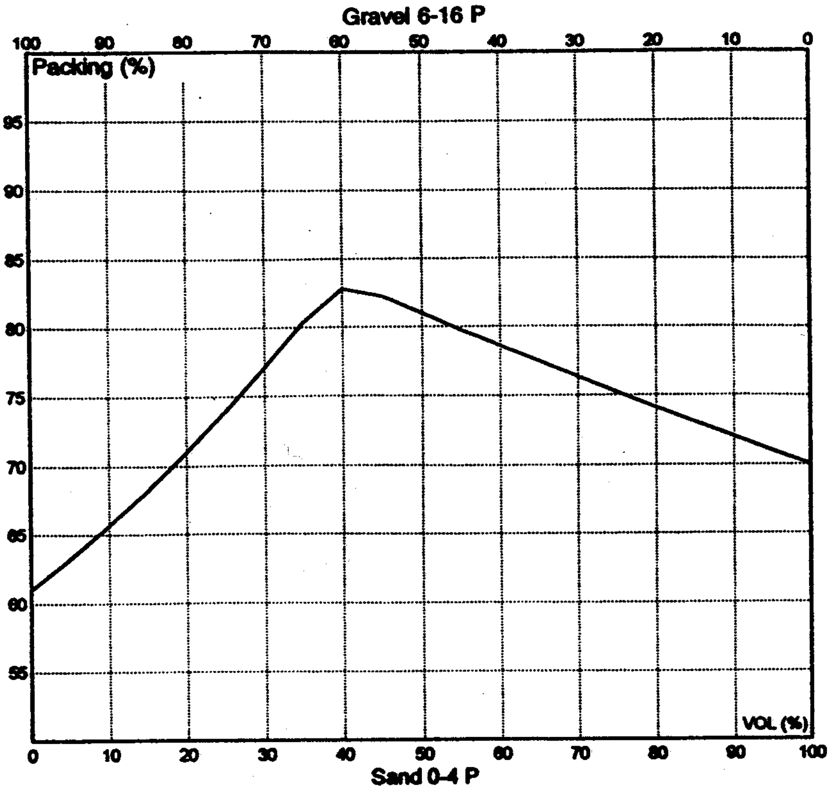

FIG. 1. *Result of a packing calculation with two different materials (μ=0.07, divisions of the grading curve=20, calculation combinations=8).*

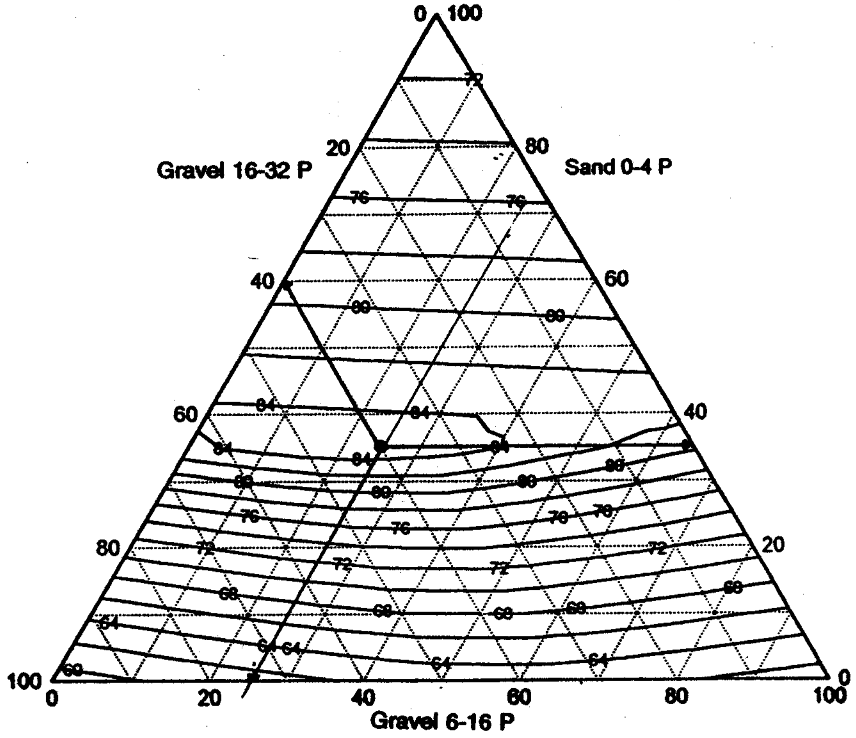

FIG. 2. *Result of a packing calculation with three different materials (μ=0.07, divisions of the grading curve=20, calculation combinations=8).*

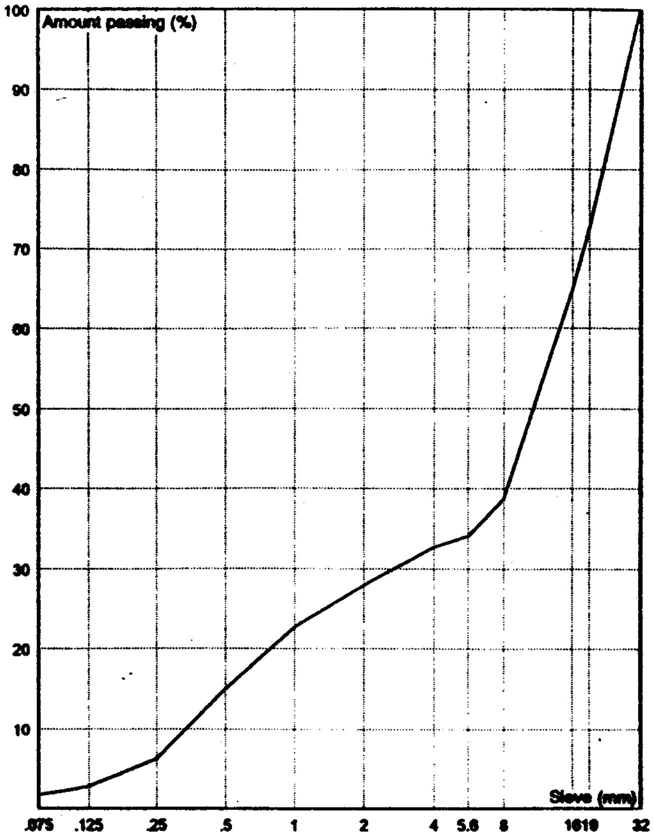

FIG. 3. *Compound grading curve for the marked point at FIG. 2 (35% Sand 0-4 P, 25% Gravel 6-16 P and 40% Gravel 16-32 P).*

4. DETERMINATION OF PACKING

No standardized method exists which is suitable for determination of the packing of aggregates. On the basis of several trial packings, it has been found most convenient to pack the aggregates in such a way that the most dense packing is achieved. This is not by vibration, but by a combined shaking-tapping process, Thygesen et al. /8/.

Determination of packing is carried out in the following way. The material is dried in an oven and a representative sample is selected. Approximately 1/3 of the sample is poured into a container. The container is tapped against a hard cover and shaked until the maximum packing is achieved. The procedure is repeated until the container is filled. Then, the surface is smoothened and the container with the material is weighed. Knowing the weight and volume of the container and the density of the material, makes it possible to calculate the packing. Three determinations are carried out and the mean value of these is the packing to use in the computer program. The procedure is described in detail in the user manual, Olsen et al. /7/.

The precision for determination of packing is approximately ± 2 %. This means that for a correct value of 0.60, an interval of 0.59 to 0.61 can be expected.

Table 2 shows typical values for experimentally determined packing and theoreticallly calculated mono-disperse packing for different Danish aggregate types. It can be seen from table 2 that the round particles (sea and soil materials) obtain a higher packing than the sharp-edged particles (granite).

Table 2. *Typical values for experimental packing and theoretical mono-disperse packing for different Danish aggregates types.*

Material	Packing	Mono-disperse packing
Sea sand	0.70	0.56
Sea stone	0.62	0.57
Soil sand	0.70	0.56
Soil stone	0.62	0.57
Granite sand	0.60	0.48
Granite stone	0.58	0.52

5. CALIBRATION AND VERIFICATION OF THE MODEL

5.1 Sensitivity analyses

Sensitivity analyses have been carried out with regard to the μ-value, the amount of dividings of the grading curve, the amount of calculation combinations and the experimental packing.

A μ-value of 0.07 is found to be valid for Danish sea aggregates, Thygesen et al. /8/, while a μ-value of 0.20 is valid for spherical particles, Stovall et al. /5/. A sensitivity analysis for the μ-value has shown that differences from 0.05 to 0.10 do not result in large variations in the packing diagram. On the basis of the above considerations, it is resonable to use a μ-value of 0.07 ± 0.03 for aggregates.

If the correct value of μ has to be determined, it can be done in the following way. The packing is determined experimentally for selected combinations of the three two-component systems which can be constructed from three materials. Then, the packing is calculated for different values of μ for the three two-component systems. The value of μ which results in the best agreement between theory and experiment is the correct μ. The mean value of the three determinations can be used in the packing calculation of the three materials.

The amount of divisions of the grading curve and the amount of calculation combinations determine the calculation accuracy and thereby also the time of calculation. Typically, the larger these parameters are, the more soft are the contour lines on the packing diagram. The parameters have to be optimized by the user in relation to the desired accuracy and the capacity of the personal computer.

Variation of the experimental packing has an influence on the resultant packing diagram too. Sensitivity analyses have been carried out for the level of precision at ±0.01, see section 4. The analysis shows that the siting of the optimum in the packing diagram does not change, while the maximum packing changes approximately 2 %.

A more detailed description of the above analysis can be seen in Glavind et al. /9/.

5.2 Verification of the model

FIG. 3. shows the result of packing calculations for three different materials. In addition, experimental determination of the packing of selected material combinations are shown with numbers together with circle sumbols. It can be seen from the figure that good agreement is obtained between theoretical and experimental values for the packing. Even though, only af few experiments have been made, the figure indicates that the model describes the packing of aggregates very well.

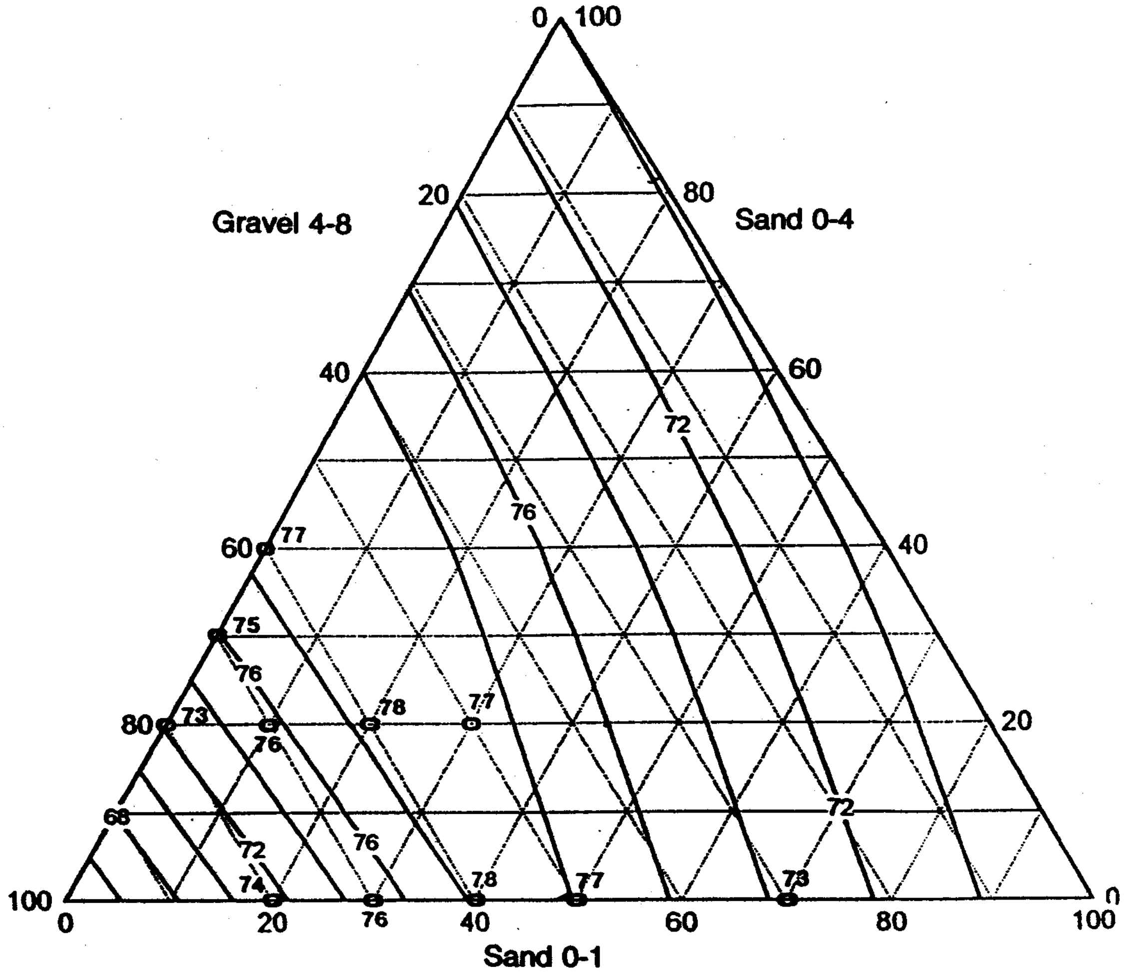

FIG. 3. *Comparison between theoretical (lines) and experimental (circles) packing determinations ($\mu=0.06$, divisions of the grading curve=10, calculation combinations=15).*

6. PACKING OF THE AGGREGATES AND PROPERTIES OF THE FRESH CONCRETE

In order to investigate the theory of a relation between the packing of the aggregates and the properties of the corresponding fresh concrete, a test serie was carried out at a Danish concrete factory.

6.1 Test program

The investigation was based in a standard mix design (15 MPa, passive environmental class). In all batches, the cement- , the water-, the air entraining additive - and the plasticizing content as well as the total volume of aggregates was kept constant. Only combinations of aggregates - and thereby the packing of the aggregates - was varied. The investigated batches are shown in the packing diagram in FIG. 4.

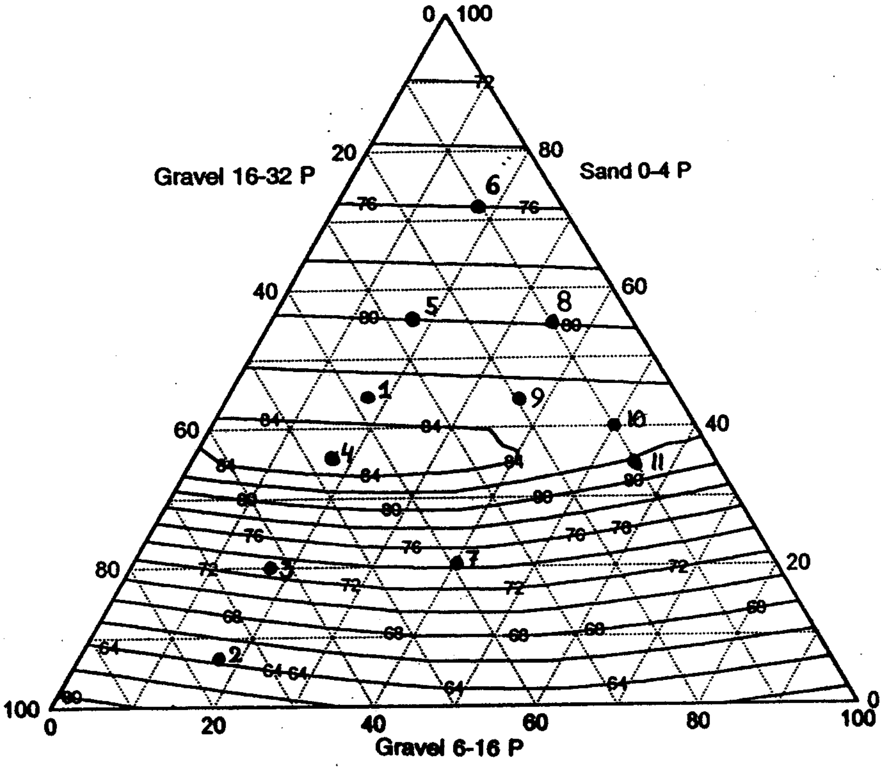

FIG. 4. *Packing diagram showing the investigated batches (μ= 0.0-7, divisions of the grading curve=20, calculation combinations=8).*

The following measurements were carried out at the fresh concrete. Density, air content, slump, vebe, flow table test, bleeding as well as a visual evaluation.

6.2 Results

The results of the investigation are illustrated most conveniently by the use of the packing diagram, see FIG. 5 and FIG. 6.

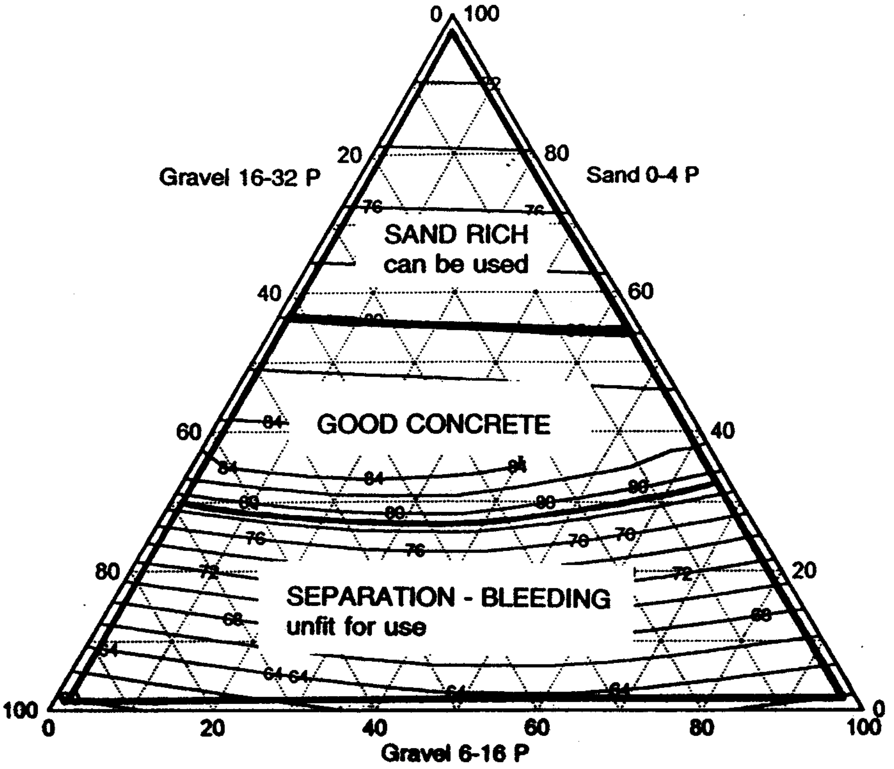

FIG. 5. *Visual evaluation of the fresh concrete (μ=0.07, divisions of the grading curve=20, calculation combinations=8).*

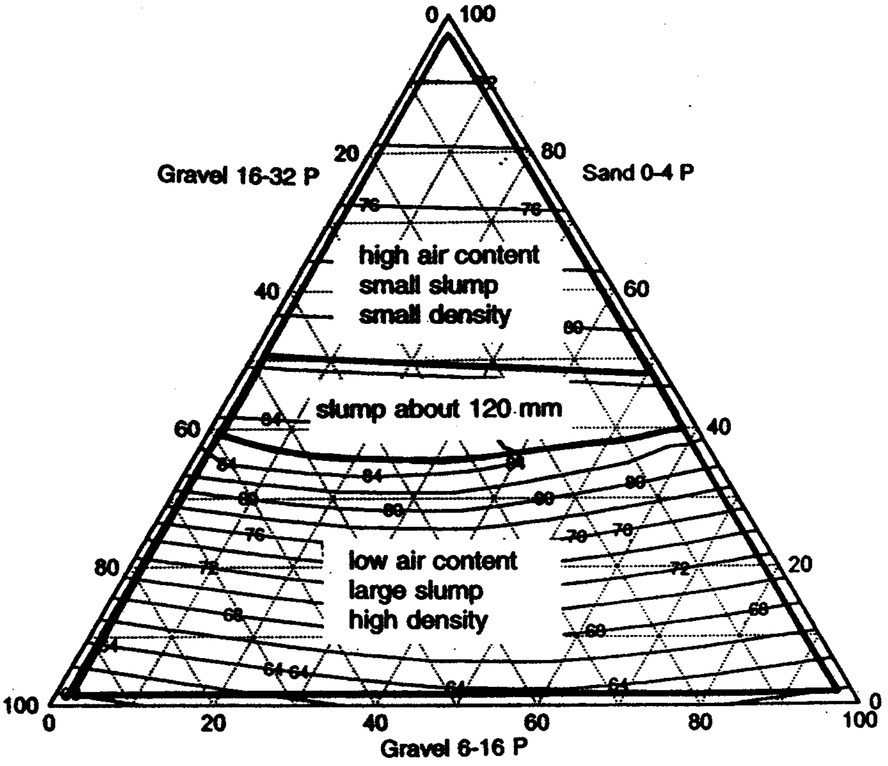

FIG. 6. *Meausured properties of the fresh concrete (μ=0.07, divisions of the grading curve=20, calculation combinations=8).*

It can be seen from the two figures, that the combinations of aggregates which result in a good and easily cast concrete are placed in - and above - the optium packing. The aggregate combinations in the lower part of the packing diagram are to be avoided. These combinations result in a more or less separated concrete with a very low air content and a large slump. The contour lines in this area are closely spaced which means that a small variation in the aggregate combination results in large variations in the packing - and thereby in the properties of the concrete.

Aggregate combinations in the upper part of the packing diagram - with a high sand content - can be used. However, the air content is very high and the slump small. The aggregate combinations which leads to a bad concrete, correspond to the areas in the packing diagram where the packing is so small that the volume of paste which is kept constant is less than the available cavities between the aggregate. This leads to a high air content for the concretes with a high sand content bacause the sand encapsulates the air. On the contrary, the concretes with a low sand content have a very low air content because the air can not be encapsulated by the aggregates.

FIG. 7 shows a photo of cylinders from batches No. 1, 2, 6 and 3. The photo illustrates very obvious the influence of the combination of the aggregates on the concrete. Especially that the concrete with a very low sand content (no. 2 and 3) and a low packing is unfit for use.

FIG. 7. *Photo of cylinders from batches No. 1, 2, 6 and 3.*

The conclusion from the investigation is that there is a relation between the packing of the aggregates and the properties of the corresponding fresh concrete. It is possible to point out areas in the packing diagram with different aggregate combinations which result in concretes with different properties. The result of the investigation is not surprising in relation to practical experience and rules of thumb, but they demonstrate the applicability of the packing calculation as a well-arranged tool for concrete mix design. The test serie is exhaustively described in Glavind et al., /10/.

7. APPLICATION POSSIBILITIES

It is important to remember that packing calculations are determined on aggregates and not on the concrete. This fact - together with among other things the inaccurcy when determining the experimental packing - explains why it is meaningless to define a point which has the best packing, but rather an area with an optimum packing, see section 6. Furthermore, it is important to bear in mind that other parameters than the packing of the aggregate influence properties of the fresh and the hardened concrete. Together with trial testing and empirical rules, the packing calculation can be useful for:

- choice of aggregate types. The experimental packing of a specific aggregate type as well as the theoretical packing of the packing of several aggregate types gives an extra material parameter apart from conventional material parameters to use when chosing an aggregate type.
- choice of aggregate combinations, which leads to the maximum packing. In addition, the packing diagram gives a survey of the sensitivity of variations in the production, i.e. the steepnes of the contour lines. Therefore, it is often reasonable to choose a higher sand content than the one which leads to the maximum packing, see section 6.
- choice of amount of paste. The amount of paste is optimized so that the volume suits the available volume between the aggregates. It means that when a higher packing is obtained, the amount of paste, including the cement, can be reduced which leads to an economic benefit, as mentioned in section 1.

The packing program has been tried-out at different concrete factories producing fresh concrete, concrete elements and different concrete products (for instance pipes). In addition, the packing program has been tried-out on concretes used for pavements. These investigations show that the aggregate combinations in well functioning concretes are placed in - or close to - the optimum packing in the packing diagram. This confirms the applicability of the packing concept.

In some cases, it has been possible to optimize the recipe, either with regard to the aggregate combination or the amount of paste.

A more detailed description of the different application possibilities can be seen in Glavind et al. /9/.

8. CONCLUSION

It has been shown that the packing of aggregates can be calculated on the basis of an experimentally determined packing, the density and the grading curve for each material. The resultant packing is a function of partly the grading curve and partly the shape of the particles.

Furthermore, a relation between the packing of the aggregates and the properties of the corresponding concrete has been demonstrated.

Packing calculations can be used for concrete mix design, choice of aggregate type and composition and to select the necessary amount of paste. Finally, the applicability of packing calculations in practical concrete production has been demonstrated.

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